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ТЕРМООПТИЧЕСКОЕ ВОЗБУЖДЕНИЕ ЗВУКА БЕССЕЛЕВЫМИ СВЕТОВЫМИ ПУЧКАМИ В СЛОИСТЫХ СРЕДАХ С ВНУТРЕННИМИ НАПРЯЖЕНИЯМИ

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THERMOOPTICAL EXCITATION OF SOUND BY BESSEL LIGHT BEAMS IN LAYERED MEDIA WITH INTERNAL STRESS

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Представлены результаты исследования фотоакустического преобразования различных мод БСП в гиротропных слоистых средах при пьезоэлектрическом методе регистрации результирующего сигнала. Показано, что амплитуда фотоакустического сигнала определяется теплофизическими характеристиками слоистых сред с внутренними напряжениями, коэффициентом поглощения и параметром кругового дихроизма гиротропных слоистых сред, поляризацией и энергетическими характеристиками БСП, частотой амплитудной модуляции, а также геометрией системы «пьезоэлектрический датчик – гиротропный слоистый образец».

Ключевые слова: *фотоакустическое преобразование, пьезоэлектрическая спектроскопия, бесселев световой пучок, диссипация энергии, функция Бесселя, уравнение теплопроводности, гиротропная слоистая среда, константы Мурнагана, коэффициенты Ламэ.*

The results of investigation of photoacoustic transformation of different modes BLB in gyrotropic layered media are submitted at piezoelectric detection method of registration of a resulting signal. Following the obtained expressions, it is showed that the amplitude of a photoacoustic signal is determined by the thermophysical characteristics of layered media with internal stress, absorption coefficient and parameter of circular dichroism of gyrotropic layered media, polarization and energy characteristics of BLB, frequency of amplitude modulation, and also by geometry system piezolelectric sensor – gyrotropic layered sample.

Keywords: *photoacoustic transformation, piezoelectric spectroscopy, Bessel light beam, energy dissipation, Bessel function, heat equation, gyrotropic layered media, Murnaghan constants, Lame coefficients.*

Introduction

The research of the mechanism of photoacoustic transformation at Bessel light beams (BLB) uptake in media with spatial dispersion is interesting from both practical and theoretical points of view. The relevant feature of BLB is the absence of diffraction divergence during their distribution. Taking that fact that natural layered gyrotropic media are widely used in quantum electronics, linear and nonlinear optics, in items of nanotechnologies, analysis of a broad spectrum of physical, acoustics, thermal etc. properties in such media is very actual.

1 Method

One of the most important line of the development of modern methods of nondestructive testing and diagnostic facilities is connected with the detection of mechanical stresses in solids. A range of methods has been developed and is being applied for the solution of this problem at present. The most important of them are as follows: ultrasound, diffraction, magnetic, and photothermoacoustic, as well as methods based on the use of holographic interferomentry. However, great attention has been recently paid to the possibility of the use of photothermoelastic effect for the diagnostics of mechanical stresses in solids [1]–[6]. An important advantage of the photothermoelastic method is its possibility to research different objects such as, for example, gyrotropic [7] and nonlinear [8] crystals, layered [9] and inhomogeneous media [10], different classes of piezo crystals symmetry [11] and so on.

Let's note that the paper [5] studied the resonant photoacoustic phenomena in mechanically stressed solids, taking into account the dependence of the thermoelastic coupling coefficient on the initial deformation in an isotropic sample upon irradiation him monochromatic plane electromagnetic waves.

Bessel light beams attract attention of researches due to their unique properties i.e. nondifraction propagation in definite region of space, as well as the ability of high concentration of light energy nearby an axis of beam [12]–[14]. The important feature of Bessel light beams is the existence of radial energy flux. The use of opportunity to manage the choice of necessary polarized modes of BLB let us to propose a device [15] of thermooptical excitation of acoustic waves with the aim to develop the method of photoacoustic diagnostics of different media and materials.

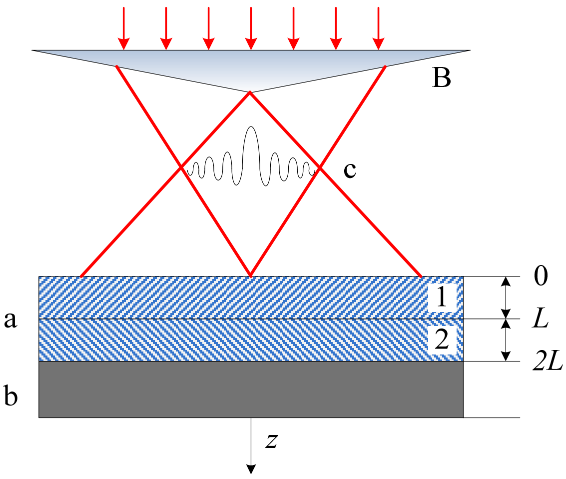

Figure 1.1. – Schematic of detection of a photoacoustic signal: (a) – sample (1) gyrotropic-isotropic layer, (2) isotropic layer; (b) piezoelectric cell, (B) conical lens, (C) modes Bessel light beams

This paper aims at studing the resonant photoacoustic transformation in a two-layer gyrotropic medium under generation of a thermoelastic signal by TE-mode of BLB (Bessel light beam). Detection of the signal is carried out using piezoelectric transducer, in the geometry shown in Figure 1.1.

2 Results

The research of photoacoustic transformation of *TE*-modes of BLB in multilayer will be described by means of material equations and Maxwell's equations.

The properties of a gyrotropic sample will be described with the use of coupling equations [16]

$$\begin{aligned} \mathbf{D} &= \varepsilon\varepsilon_0\ \mathbf{E} + i\eta\sqrt{\varepsilon_0\mu_0}\,\mathbf{H}, \\ \mathbf{B} &= \ \mu\mu_0\mathbf{H} - i\tilde{\eta}\sqrt{\varepsilon_0\mu_0}\,\mathbf{E}, \end{aligned} \tag{2.1}$$

where and $\mu_0 = 4\pi\cdot 10^{-7}\ H\cdot m^{-1}$ are electric and magnetic constants respectively, μ is magnetic medium permittivity, $\mu = 1$ – it is assumed that the medium is nonmagnetic, the sign '~' is used to denote transposition, η is a pseudoscalar complex parameter of gyrotropy, and the real part $\mathrm{Re}\,\eta = \eta_1$ defines specific rotation of the plane of polarization, and the imaginary part $\mathrm{Im}\,\eta = \eta_2$ is responsible for circular dichroism.

The energy dissipation in the gyrotropic sample can be found basing on (2.1) and Maxwell equations and can write down as follows

$$Q^{TE} = \tilde{Q}^{TE} e^{-2k_{z2}z},$$

$$\tilde{Q}^{TE} = \frac{\omega|\varepsilon|\varepsilon_2\varepsilon_0}{2\pi}\left(\left(\frac{m}{q\rho}\right)^2 J_m^{\,2}(q\rho) + J_m'^{\,2}(q\rho) + \right. \\ \left. + \frac{2mk_0k_{z1}\eta_2}{q^3\rho} J_m(q\rho)J_m'(q\rho)\right),$$

where $k_0 = \omega/c$, ε is the medium permittivity, ρ is a radial coordinate, $q = k_0\sqrt{\varepsilon}\sin(\gamma)$, $k_{z1} = k_0\sqrt{\varepsilon_1}\cos(\gamma_1)$, $k_{z2} = k_0\sqrt{\varepsilon_2}\cos(\gamma_2)$, $\varepsilon_1 = \varepsilon_1' + i\varepsilon_1''$ is a complex permittivity of the gyrotropic-isotropic layer, $\varepsilon_2 = \varepsilon_2' + i\varepsilon_2''$ is the complex permittivity of the second isotropic layer, γ_1 и γ_2 are conicity parameters of BLB in the first and the second layer, which is equal to the half apex angle of the cone of the wave vectors that define the spatial frequency spectrum of the beam, ω_0 is the frequency of light, ω is the frequency of amplitude modulation of BLB, $J_m(q\rho)$ is a m-order Bessel function of the first kind, $J_m'(q\rho) = \partial J_m(q\rho)/\partial\rho$.

Basing on the formerly used method in [5] we obtain the expression for the photoacoustic (PA) response from the piezoelectric transducer under generation of a thermoelastic signal in the layered medium (here, a double layer) of BLB when the boundaries are free:

$$V = \frac{\begin{pmatrix} iY_u(2L)\times \\ \times\left(c_u^T Q_u 2\cos(Q_{\text{г}}L) - c_{\text{г}}^T Q_{\text{г}} i2\sin(Q_{\text{г}}L)\right) - \\ -iX_1\left(c_u^T Q_u 2\cos(Q_{\text{г}}L) - c_{\text{г}}^T Q_{\text{г}} 2i\sin(Q_{\text{г}}L)\right) - \\ -c_u^T C e^{iQ_u L} \end{pmatrix}}{\begin{pmatrix} c_u^T\left(e^{i2k_1L_1} + 1\right)\times \\ \times\left(c_u^T iQ_u 2\cos(Q_{\text{г}}L) - c_{\text{г}}^T iQ_{\text{г}} 2i\sin(Q_{\text{г}}L)\right) + \\ + c^D k_1\left(1 - e^{i2k_1L_1}\right)\times \\ \times\left(c_u^T Q_u 2\cos(Q_{\text{г}}L) - c_{\text{г}}^T Q_{\text{г}} 2i\sin(Q_{\text{г}}L)\right) \end{pmatrix}} \times \\ \times\left(2e^{ik_1L_1} - e^{ik_1 2L_1} - 1\right)$$

with the following designations:

$$X_0 = c_{\text{г}}^T \frac{\partial Y_{\text{г}}(z)}{\partial z}\Big|_{z=0} - B_{\text{г}}\alpha_{t\text{г}}T_{\text{г}}(0),$$

$$X_1 = c_u^T \frac{\partial Y_u(z)}{\partial z}\Big|_{z=2L} - B_u\alpha_{tu}T_u(2L),$$

$$X_2 = Y_{\text{г}}(L) - Y_u(L),$$

$$X_3 = \begin{pmatrix} c_{\text{г}}^T \dfrac{\partial Y_{\text{г}}(z)}{\partial z}\Big|_{z=L} - c_u^T \dfrac{\partial Y_u(z)}{\partial z}\Big|_{z=L} + \\ + B_u\alpha_{tu}T_u(L) - B_{\text{г}}\alpha_{t\text{г}}T_{\text{г}}(L) \end{pmatrix},$$

$$C = c_u^T\left(X_3 - X_0e^{iQ_{\text{г}}L}\right)\left(e^{-iQ_{\text{г}}L} + e^{iQ_{\text{г}}L}\right) + \\ + \left(e^{iQ_{\text{г}}L}X_0 - c_{\text{г}}^T iQ_{\text{г}}X_2\right)\left(e^{iQ_{\text{г}}L} - e^{-iQ_{\text{г}}L}\right),$$

$$T_{\text{г}}(z) = B_1e^{-\sigma z} + B_2e^{\sigma z} + \psi_{\text{г}}e^{-2k_{z2\text{г}}z},$$

$$T_u(z) = B_3e^{-\sigma z} + \psi_u e^{-2k_{z2u}z},$$

$$B_1 = \frac{(\phi_1 + \sigma\phi_2)e^{-\sigma L} - 4k_{z2г}\psi_г}{2\sigma},$$

$$B_2 = \frac{(\phi_1 + \sigma\phi_2)e^{-\sigma L}}{2\sigma},$$

$$B_3 = \frac{(\phi_1 + \sigma\phi_2)\left(e^{-\sigma L} + e^{\sigma L}\right) - 4k_{z2г}\psi_г}{2\sigma} - \phi_2 e^{\sigma L},$$

$$\psi_г = \frac{\tilde{Q}_г^{TE}}{2k_{sг}\left(\sigma^2 - 4k_{z2г}^2\right)}, \quad \psi_u = \frac{\tilde{Q}_u^{TE}}{2k_{su}\left(\sigma^2 - 4k_{z2u}^2\right)},$$

$$\sigma = \sqrt{\frac{i\omega}{\beta}},$$

$$\phi_1 = \left(2k_{z2г}\psi_г e^{-2k_{z2г}L} - 2k_{z2u}\psi_u e^{-2k_{z2u}L}\right),$$

$$\phi_2 = \left(\psi_u e^{-2k_{z2u}L} - \psi_г e^{-2k_{z2г}L}\right),$$

$$Y_г(z) = Y_1 e^{-\sigma z} + Y_2 e^{\sigma z} + Y_3 e^{-2k_{z2г}z},$$

$$Y_u(z) = Y_4 e^{-\sigma z} + Y_5 e^{-2k_{z2u}z},$$

$$Y_1 = -\frac{g^{(3)г}\sigma B_1}{G_{3г}^{(3)}\left(\sigma^2 + Q_г^2\right)}, \quad Y_2 = \frac{g^{(3)г}\sigma B_2}{G_{3г}^{(3)}\left(\sigma^2 + Q_г^2\right)},$$

$$Y_3 = -\frac{g^{(3)г}2k_{z2г}}{G_{3г}^{(3)}\left(4k_{z2г}^2 + Q_г^2\right)}, \quad Y_4 = -\frac{g^{(3)u}\sigma B_3}{G_{3u}^{(3)}\left(\sigma^2 + Q_г^2\right)},$$

$$Y_5 = -\frac{g^{(3)u}2k_{z2u}}{G_{3u}^{(3)}\left(4k_{z2u}^2 + Q_u^2\right)},$$

$$Q_г^2 = \frac{\rho_{0г}\omega^2}{G_{3г}^{(3)}}, \quad Q_u^2 = \frac{\rho_{0u}\omega^2}{G_{3u}^{(3)}},$$

$$G_3^{(3)} = t_{33}^{(0)} + b + 2(n + m_0)U_{33} + C_{33},$$

$$b = 2\mu + (2m - n)U_{33},$$

$$C_{33} = K - \frac{2}{3}\mu + 2L_0U_{33}, \quad g^{(3)} = \left(1 + \vartheta U_{33}\right)\gamma_0,$$

ρ_0 is the sample density at the initial instant; ϑ is the coefficient determining the dependence of the thermoelastic coupling on the initial strain; γ_0 is the thermoelastic coupling coefficient for the unstrained sample; K is the compressibility; m_0, n, L_0 are the Murnaghan constants; U_{33} is a component of the initial deformation vector; $t_{33}^{(0)}$ is a component of the initial stress tensor; $h = e/\varepsilon^s$; e is a piezoelectric modulus; ε^s is the permittivity of clamped crystal; $c^D = c^E\left(1 + e^2/\left(\varepsilon^s c^E\right)\right)$; c^E is the coefficient of piezoelectric stiffness; $c^T = \lambda + 2\mu$, λ and μ are the Lame coefficients; B is the bulk elasticity modulus; α_t is the coefficient of thermal volume expansion; k_s is the thermal conductivity; β is the thermal diffusivity; index $г$ corresponds to the gyrotropic-isotropic layer; index u corresponds to the isotropic layer.

For the simplicity of further calculations, let us choose the media under consideration such that they have approximately identical thermophysical parameters.

3 Graphical analysis

As seen from the graphs (Figure 3.1), for TE mode of the function $Q^{TE}(\rho)$ for BLB_0 and oscillate nearly in antiphase. This peculiarity is conditioned by the out-of-phase component S_φ^{TE}.

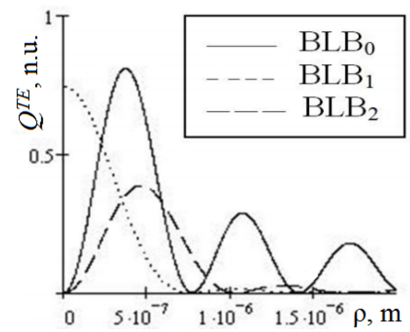

Figure 3.1. – Radial distribution of the energy dissipation $Q^{TE}(\rho)$ for different BLB

Change the shape of the graphs for $Q^{TE}(\rho)$ at BLB_1 and BLB_2 of TE-polarization occurs as a result of the combined influence of the two factors: the emergence of the azimuthal energy flux S_φ^{TE} and the absence of the radial flow S_ρ^{TE} that determine the diffraction spreading of a light beam near of paraxial area.

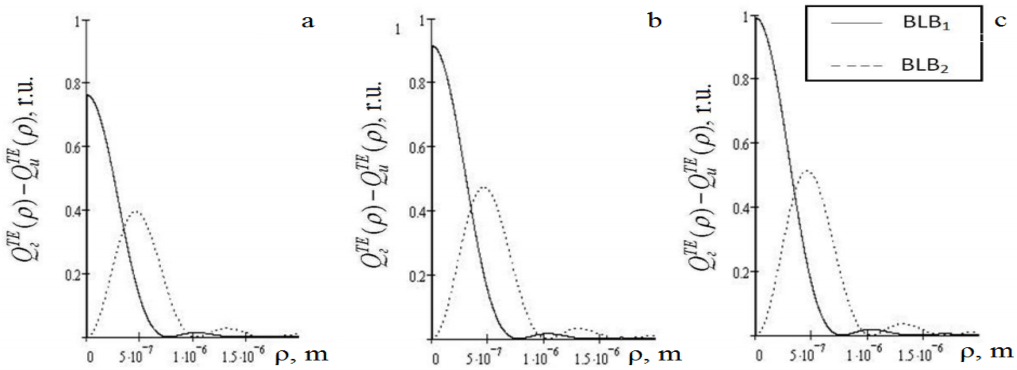

Figure 3.2. – Dependence of the difference of energy dissipation from the ρ as the parameter of gyrotropy η_2 $Q_г^{TE}(\rho)$ and $Q_u^{TE}(\rho)$: (a) $\eta_2 = 10^{-5}$; b) $\eta_2 = 1.2\cdot10^{-5}$; (c) $\eta_2 = 1.3\cdot10^{-5}$

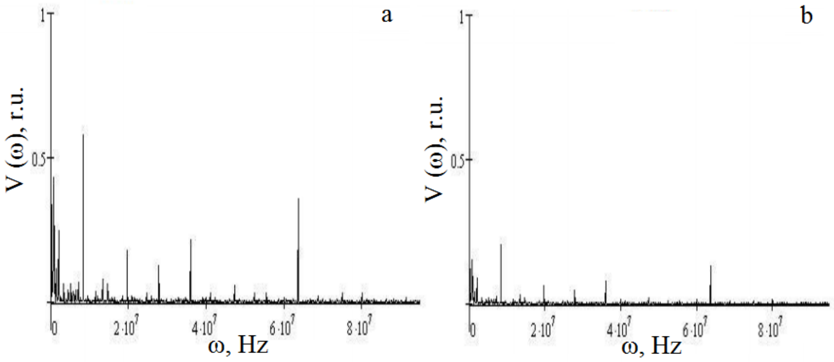

Figure 3.3. – Dependence of the photoacoustic signal from frequency modulation BLB_1 for different ρ: (a) $\rho = 10^{-7}$ m; (b) $\rho = 4\cdot10^{-7}$ m

For gyrotropic absorbing medium (Figure 3.2), one can see the notable difference in radial distribution of the heat dissipation

$$Q^{TE}(\rho) = Q_e^{TE}(\rho) - Q_u^{TE}(\rho),$$

it is related to the imaginary part of the gyrotropy parameter, which is responsible for circular dichroism, therefore, the increase parameter of gyrotropy η_2 leads to growth in the absorbed energy of BLB.

The curve (Figure 3.3) shows that the amplitude of the photoacoustic response depends on the frequency of the amplitude modulation. As follows from the graphs (Figure 3.1), the maxima of amplitudes of PA signal are defined by the resonant properties of the system 'layered sample – piezoelectric transducer', namely by the relation of geometric parameters of the investigated sample and piezodetector, by the Murnaghan constants and Lame coefficients.

Conclusions

Thus, the following results were obtained in the paper:

1. The rate of change of energy dissipation under absorption of BLB in the double-layer gyrotropic medium was calculated. The dependence of the dissipation of TE mode of BLB on the radial coordinate and the value of the imaginary part of gyrotropy parameter were analyzed. It was graphically shown that the energy dissipation can change considerably under the change parameters of the gyrotropy.

2. The amplitude-phase characteristics of the PA signal, which occurs in the double-layer medium with internal stress, were defined taking into account the thermoelastic coupling coefficient. The dependence of the amplitude of the PA response on the modulation frequency and radial coordinate was graphically analyzed. The resonant increase in the amplitude of the PA signal for the frequencies $\omega > 100\ KHz$ of the incident double-layer BLB sample was revealed. It was shown that the maxima of the amplitude of the PA signal correspond to the values of radial coordinates, near which the maxima of energy dissipation are localized.

3. There by, expression for amplitude of photoacoustic signal in mechanically stressed double layer was obtained with due regard to dependence of thermoelastic connection coefficient on initial deformation in sample. It should be noticed that experimental amplitude measurement of photoacoustic signal for different polarizing BLB will suggest (on the ground of obtained expressions) the method of definition of thermoelastic coupling coefficient as well as Murnaghan constants in crystal medium containing internal stress.

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